

Overcome Limited Data Challenge in Time Series Forecasting with Power Law Algorithm for Attribution Churn Value

Cindy Hapsari¹, Bagus Gede Krishna Yudistira²

^{1,2} Jurusan Teknologi Informatika, Fakultas Teknik dan Kejuruan, Universitas Pendidikan Ganesha, Singaraja, 81116, Indonesia

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ABSTRAK

Keterbatasan data historis merupakan salah satu hambatan utama dalam analisis churn dan atribusi nilai produk berbasis deret waktu. Penelitian ini mengusulkan penggunaan Algoritma Power Law sebagai pendekatan alternatif untuk melakukan estimasi perilaku churn pada kondisi data minimal. Dengan menggunakan data dummy yang merepresentasikan nilai produk dari beberapa judul film, metode ini mengevaluasi kemampuan Power Law dalam memodelkan pola perubahan nilai konsumen serta kecenderungan churn pada horizon harian. Algoritma diterapkan melalui proses fitting nonlinear dan estimasi parameter skala untuk menangkap hubungan antara frekuensi dan besaran nilai yang diamati. Hasil penelitian menunjukkan bahwa Power Law mampu menghasilkan pola prediksi yang stabil dan konsisten meskipun data yang tersedia sangat terbatas, serta memberikan gambaran awal mengenai dinamika kontribusi nilai suatu konten terhadap risiko churn pelanggan. Temuan ini memberikan dasar bagi pengembangan pendekatan prediksi churn yang lebih efisien, khususnya pada lingkungan bisnis yang memiliki keterbatasan data historis.

ABSTRACT

Limited historical data poses a significant challenge in time series forecasting for churn analysis and product value attribution. This study introduces the Power Law Algorithm as an alternative approach to estimate churn-related behavioral patterns under minimal data availability. Using dummy data representing product value metrics from a set of film titles, the method evaluates the capability of Power Law modeling to capture shifts in consumer value and daily churn tendencies. The algorithm is applied through nonlinear curve fitting and scale-parameter estimation to identify the underlying relationship between frequency and magnitude within the observed values. The results indicate that the Power Law approach can produce stable and consistent prediction patterns even with highly constrained datasets, offering an initial analytical foundation for understanding how content value contributes to customer churn risk. These findings highlight the potential of the Power Law Algorithm to support more efficient churn forecasting frameworks, especially in environments where comprehensive historical data is not available.

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*Penulis Koresponden

Email: cindy@student.undiksha.ac.id

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1. BACKGROUND

Churn analysis and product value attribution play a critical role in subscription-based industries, particularly in digital content platforms such as video-on-demand services. Understanding how individual content titles contribute to customer retention or churn risk is essential for informed strategic decision-making, including content investment planning, user segmentation, and marketing budget allocation. However, churn forecasting models typically require extensive historical time series data to produce accurate and dependable predictions. In operational settings, such rich historical data are often unavailable due to factors such as newly released content, rapid shifts in user consumption behavior, or internal limitations in data collection systems. This creates a need for mathematical and algorithmic approaches capable of estimating churn-related patterns effectively, even when only minimal data are available.

The Power Law Algorithm offers a promising solution to this challenge. Mathematically, Power Law distributions are known for modeling scale-dependent and nonlinear phenomena, including content consumption patterns, user interaction frequency, and retention dynamics. Its ability to capture relationships between magnitude and occurrence frequency makes it particularly relevant for attribution churn economics, where the economic value of a film title evolves over time and influences a customer's likelihood to remain subscribed. By utilizing dummy data that represent product value metrics, this study investigates the feasibility of applying Power Law modeling to daily churn-related forecasting under severe data constraints.

This approach has the potential to contribute in two significant ways at the first, by providing an adaptive early-stage estimation technique suitable for scenarios with limited data and second, by offering mathematical insights into how content value dynamics affect churn risk. The findings are expected to lay a methodological foundation for the development of more efficient churn forecasting systems, particularly in business environments that demand rapid decision-making despite limited historical data availability.

Although the power law approach has been widely used in various forecasting and modeling studies on economic dynamics, its application is still dominated by relatively data-rich contexts and focuses on short-to medium-term risk analysis or statistical behavior. Studies on agent-based economic forecasting models show that prediction accuracy is highly dependent on the availability of highly detailed micro and macro data, making it less applicable to business scenarios with limited historical data[1].

On the other hand, research on power law trends in speedrunning and machine learning benchmarks has successfully demonstrated the power law's ability to predict with a limited number of observations. However, its primary focus remains on patterns of technical performance improvement, rather than long-term business forecasting needs[2]. Meanwhile, studies on company growth prediction based on scaling law and econophysics-informed models emphasize the power law's superiority in capturing average growth patterns and fluctuations[3]. However, these generally require relatively long time series and have not explicitly evaluated the reliability of predictions over very long horizons (5–8 years) with minimal data. Thus, there is a significant research gap regarding the development and evaluation of power law models specifically designed for long-term business forecasting under data constraints, both in terms of predictive stability, strategic relevance, and applicability to business decision-making. This study aims to fill this gap by examining the effectiveness of the power law approach as a robust long-term forecasting model in data-limited scenarios.

2. METHODS

This research develops a daily attribution value forecasting model that combines deterministic and stochastic approaches. This model is designed to capture both the natural decay trend and the realistic variations that occur in real data. In general, the applied methodology can be divided into several main stages:

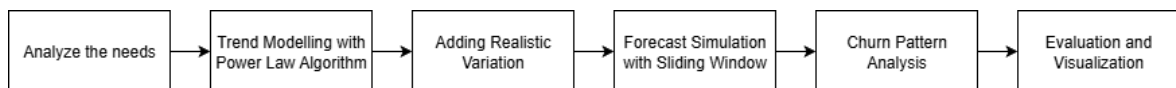


Figure 1. Flowchart of The Model

2.1. Analyze The Needs

Power-law models have been shown to effectively capture non-linear trends in time-series data across multiple domains, including agricultural yield analysis and financial market dynamics [4], [5]. Their ability to model long-term growth and variance patterns makes them suitable for trend forecasting under limited data conditions, particularly in business-related attribution and churn value analysis.

2.2. Trend Modelling with Power Law Algorithm

The power law model is well-suited for capturing long-term temporal dynamics and scale-invariant patterns in attribution value evolution, especially when data are limited and do not exhibit strong seasonality. AI trend modeling prediction itself can be used primarily focusing on integrating technical predictions with actual business decisions[6]. Additionally, integrating power law trend modelling with AI-driven churn and lifetime value frameworks provides a flexible analytical foundation for business forecasting and decision support. This aligns with recent research highlighting the role of advanced AI methods in churn prediction and lifecycle value management, which emphasize interpreting early trend signals and incorporating them into broader analytical workflows for marketing and retention optimization.

$$y(t) = C \cdot t^{-a} \quad (1)$$

- Where y represents the predicted value at time,
- C is a scaling constant that adjusts the amplitude of the curve, and
- t^{-a} is the scaling exponent that determines the rate of decay over time

The parameters C and a are estimated by nonlinear curve fitting using the Levenberg-Marquardt algorithm implemented in the `curve_fit` function of the SciPy library. To anticipate nonlinear fitting failures, a fallback mechanism is implemented by performing linear regression on the logarithmically transformed data:

$$\log(y) = \log(C) - a \cdot \log(t) \quad (2)$$

2.3. Adding Realistic Variation

The baseline trend obtained from power-law fitting is enhanced by introducing realistic variation components to reflect daily fluctuations in real-world data. A random walk with mean reversion is applied to model stochastic daily changes while ensuring convergence toward the baseline trend, controlled by a recovery speed parameter.

Weekly seasonality is incorporated to capture cyclical user behavior across different days, and occasional market shocks are introduced to represent sudden external events such as campaigns or negative influences. To ensure stability and plausibility, the generated values are constrained within predefined bounds relative to the baseline trend, and extreme daily changes are smoothed using weighted averaging. Finally, a Savitzky–Golay filter is applied to smooth the overall signal while preserving the underlying trend structure.

2.4. Forecast Simulation with Sliding Window

Forecasting is performed using a sliding window approach. At each simulation step (daily), the model is fit using data from the most recent window, which in this test used 7 days. The fitting results are used to predict the next day's baseline value.

The predicted value is then processed using a realistic variation mechanism to produce more realistic daily values. This process is repeated until the desired forecast horizon is reached (e.g., 10 years into the future). This allows the model to dynamically adjust parameters based on the most recent data, capturing any trend changes.

2.5. Churn Pattern Analysis

To identify periods where attribution value experienced a significant decline, a churn analysis was performed. A period was defined as churned if its 7-day moving average fell below 5% of its initial value. Furthermore, the daily churn probability was calculated as a function of the decline in value relative to the initial value.

2.6. Evaluation and Visualization

The simulation results are evaluated using multiple statistical metrics, including mean, standard deviation, cumulative value, peak value, and recovery rate. To facilitate interpretation, the forecasting outcomes are visualized through several representations, such as comparisons between the baseline trend and realistic simulations over the initial period, long-term trend analysis using logarithmic scaling with smoothed trend lines, cumulative attribution growth, distributions of daily changes, and tabular comparisons across different scenarios or data categories. Sample daily sequences are also presented to illustrate realistic variability.

To assess estimation stability and forecasting quality, standard error-based metrics are employed, including Mean Absolute Error (MAE) to measure average deviation, Root Mean Square Error (RMSE) to capture sensitivity to large errors, Symmetric Mean Absolute Percentage Error (SMAPE) to evaluate overall

forecasting accuracy, and Mean Absolute Percentage Error (MAPE) to quantify average relative prediction error.

3. RESULT AND DISCUSSIONS

This section presents the results and discussion of the proposed approach, covering the dataset characteristics, the pseudocode implementation, prediction outcomes, and evaluation metrics. The analysis focuses on how the power-law-based trend modeling performs under limited data conditions, highlighting both quantitative performance and qualitative behavior observed in the forecasting results. Through comprehensive metric evaluation and visual analysis, this section provides insights into the robustness and practical relevance of the proposed method.

3.1. The Resource Dataset

The dataset is used as the initial input for developing and evaluating the daily attribution value forecasting model. It consists of two controlled data variants, referred to as *Low-Intensity Attribution (LIA)* and *High-Intensity Attribution (HIA)*, each representing distinct attribution dynamics. For both variants, the dataset includes the number of days, daily attribution values, and cumulative attribution values. The daily attribution values reflect fluctuating day-to-day contributions, while the cumulative values represent the total contribution accumulated up to a given time point. The two attribution variants exhibit contrasting patterns where the LIA series is characterized by relatively small and stable values, whereas the HIA series displays substantially higher initial values with a decreasing trend over time. These controlled variations provide diverse data characteristics that are essential for evaluating the model's ability to capture long-term trends, changes in growth rates, and cumulative dynamics in a realistic forecasting setting.

Table 1. The Dataset in LIA and HIA

Days	Low-Intensity Attribution (LIA)		High-Intensity Attribution (HIA)	
	Daily Attribution	Accumulative Attribution	Daily Attribution	Accumulative Attribution
1	13	13	250	250
2	15	28	185	435
3	7	35	150	585
4	3	38	100	685
5	9	47	110	795
6	11	58	95.5	890.5
7	12.5	70.5	75	965.5
8	8	78.5	55	1025.5

3.2. Architecture Model

Table 2. Pseudocode of Architecture Power Law Forecast Model

```

FUNCTION simulate_realistic_forecast(title_data, title_name):
  days = title_data['Days']
  values = title_data['Attribution Value per Days']

  // Initialize window
  actual_window = min(window_size, len(days))
  current_days = days[:actual_window]
  current_values = values[:actual_window]

  all_days = list(days)
  all_base_values = list(values)
  all_realistic_values = list(values)
  parameters_history = []

  target_days = forecast_years * 365
  current_max_day = max(days)

  WHILE current_max_day < target_days:
    // Fit model to current window
    C, alpha, error = fit_power_law(current_days, current_values)

    // Save parameters
    parameters_history.append({
      'day': current_max_day,
      'C': C,

```

```

'alpha': alpha
})

// Predict next day
next_day = current_max_day + 1
next_base_value = power_law_func(next_day, C, alpha)

// Generate realistic value with variation
next_realistic_value = generate_next_realistic_value(
    current_values,
    next_base_value,
    title_name
)

// Update data
all_days.append(next_day)
all_base_values.append(next_base_value)
all_realistic_values.append(next_realistic_value)

// Slide window
current_days = append(current_days[1:], next_day)
current_values = append(current_values[1:], next_realistic_value)
current_max_day = next_day

// Add realistic variations to the entire series
final_realistic_values = add_realistic_variations(
    all_base_values,
    all_days,
    title_name
)

// Calculate cumulative values
cumulative_values = cumulative_sum(final_realistic_values)

// Analyze churn and statistics
churn_info = analyze_churn_pattern(final_realistic_values, all_days)
summary_stats = calculate_summary_stats(final_realistic_values, all_days)

```

The proposed pseudocode describes a forecasting simulation algorithm aimed at generating long-term projections that are both mathematically robust and representative of realistic real-world dynamics. In contrast to conventional extrapolation techniques that tend to produce overly smooth and idealized trends, this approach adopts an adaptive sliding window strategy to iteratively update model parameters using the most recent historical observations. The underlying baseline trend is modeled through a power-law formulation, enabling the capture of non-linear growth and decay patterns frequently observed in digital content consumption and market-driven products.

To enhance realism, stochastic variations conditioned on historical volatility and entity-specific characteristics are incorporated, resulting in daily forecasts that reflect natural fluctuations. At each iteration, newly generated observations are appended to the time series, the analysis window is advanced, and model parameters are recalibrated until the predefined forecasting horizon is reached. Post-simulation, additional components such as seasonal effects and sporadic events are applied, followed by cumulative value computation and descriptive statistical analysis. This hybrid framework effectively combines trend estimation with realistic variability, thereby producing more informative and actionable forecasts for long-term strategic decision-making.

First, the algorithm initializes a sliding window using the available historical data, consisting of daily time indices and corresponding attribution values. The window size is adjusted to the available data length and serves as the initial subset for model fitting. All observed data are copied into extended arrays to store baseline predictions, realistic forecasts, and parameter histories throughout the simulation.

Second, the algorithm enters an iterative forecasting loop that runs until the predefined forecasting horizon is reached. At each iteration, a power-law model is fitted to the current window to estimate the scaling parameters C and α , which are recorded for temporal analysis. The fitted model is then used to predict the baseline attribution value for the next day.

Third, the baseline prediction is transformed into a realistic forecast by injecting stochastic variations based on recent values and entity-specific characteristics. The newly generated value is appended to the time series, and the sliding window is advanced by removing the oldest observation and including the latest forecasted value. This adaptive update enables continuous recalibration of model parameters as new information becomes available.

Finally, after the forecasting horizon is reached, additional realistic variations are applied to the entire series to incorporate seasonality and sporadic events. The cumulative attribution values are then computed, followed by churn pattern analysis and descriptive statistical evaluation. These steps provide quantitative insights into long-term trends, volatility behavior, and recovery dynamics captured by the proposed forecasting framework.

4. CONCLUSION

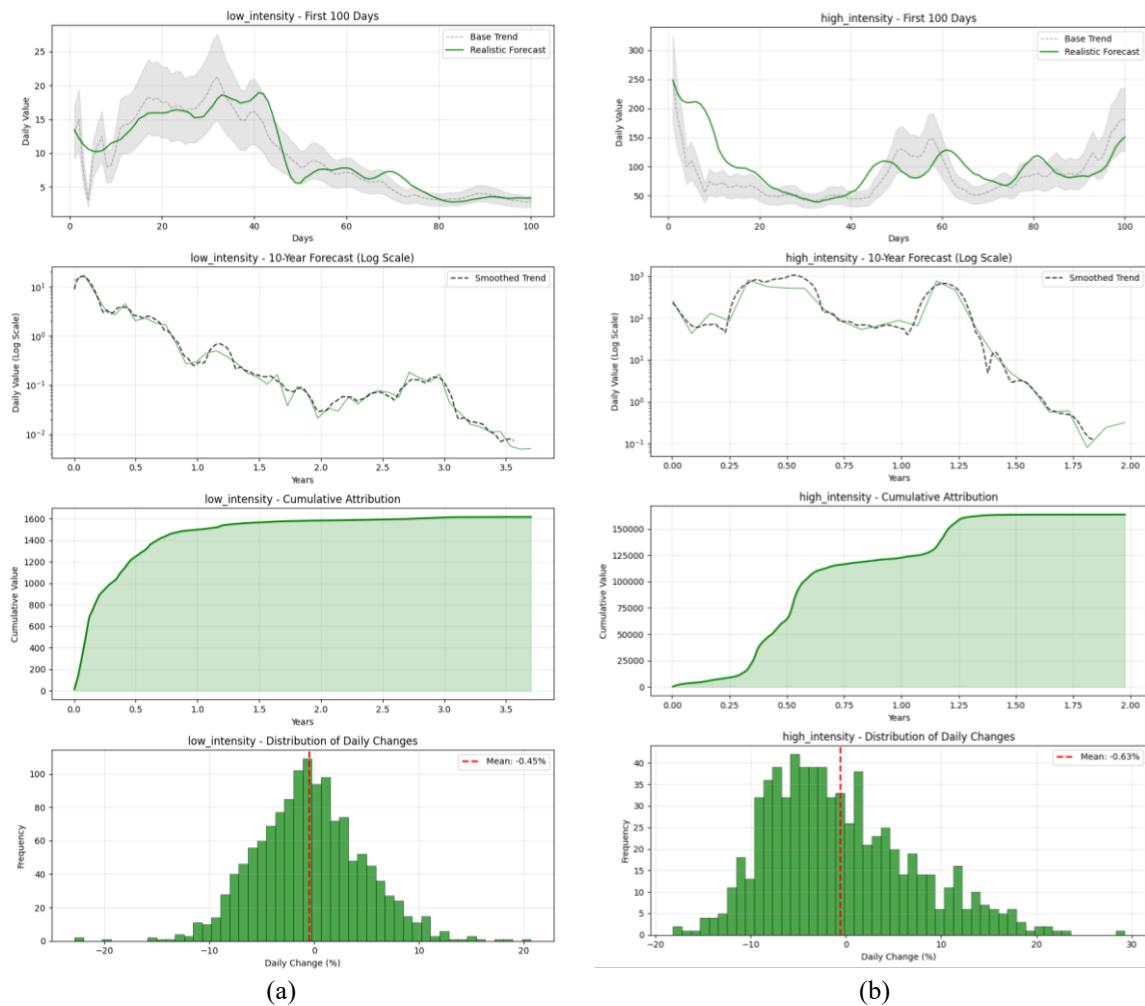


Figure 2. First 100 Days Prediction after 7 Days Data, 10 Year Forecast in Log Scale, Cumulative Attribution, Distribution of Daily Changes in Forecasted Model
 (a) Low-Intensity Attribution (b) High-Intensity Attribution

The results quantitatively demonstrate clear differences between the two intensity scenarios. The base trend of the high-intensity scenario reaches 250.0, which is ten times higher than the low-intensity base trend of 25.0. This disparity is preserved in the realistic forecasts over the first 100 days, where the average forecasted value for the high-intensity case is approximately 200.0, compared to 15.0 for the low-intensity scenario, indicating a 13.3-fold difference. However, under long-term projection using a logarithmic scale, both scenarios converge to a similar value of approximately 0.05, suggesting that the power-law model captures a common stabilization behavior regardless of initial scale.

From a cumulative perspective, the high-intensity scenario yields a total attribution value of around 150,000, significantly exceeding the low-intensity cumulative value of approximately 16,000, corresponding to a 9.375-fold increase. Despite these substantial differences in magnitude, the distributions of daily percentage changes exhibit similar ranges, with both scenarios reaching comparable extreme values of approximately -20%, indicating consistent volatility characteristics across different intensity levels. These findings confirm that the proposed model effectively differentiates scale effects while maintaining realistic and stable temporal dynamics.

Table 3. Evaluation Metrics Result in Power Law Forecasting Model

Data	RMSE	MAE	MAPE	SMAPE
Low-Intensity Attribution	3.8519	2.8772	55.92%	32.04%
High-Intensity Attribution	49.5750	45.2215	49.52%	36.89%

Based on the evaluation results, the model exhibits distinct performance characteristics across different attribution intensity levels. For the Low-Intensity Attribution scenario, the RMSE (3.85) and MAE (2.88) remain relatively low, indicating moderate absolute prediction errors; however, the MAPE (55.92%) and SMAPE (32.04%) are comparatively high due to the strong influence of daily fluctuations on small attribution values. In contrast, the High-Intensity Attribution scenario shows substantially larger absolute errors, with RMSE and MAE reaching 49.58 and 45.22, respectively, reflecting the higher data scale and increased complexity of the underlying dynamics. Nevertheless, the relative error metrics for this scenario are lower, with MAPE at 49.52% and SMAPE at 36.89%, suggesting more stable predictive performance when evaluated relative to the magnitude of the data.

These findings indicate that model performance is strongly influenced by data scale and volatility characteristics. While absolute error metrics increase with higher attribution intensity, relative accuracy improves, highlighting the suitability of the proposed approach for modeling large-scale attribution dynamics under limited data conditions.

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